Production Function and Cost Curves

Problem Set

1. Bernie's ice-making company produces ice cubes using a 10-ton machine and electricity. The quantity of output, measured in terms of pounds of ice, is given in the accompanying table.

Quantity of Electricity (kilowatts)	Quantity of Ice (pounds)
0	0
1	1,000
2	1,800
3	2,400
4	2,800

   a. What is the fixed input? What is the variable input?

   The fixed input is the 10-ton machine, and the variable input is electricity.

   b. Construct a table showing the marginal product of the variable input. Does it show diminishing returns?

   As you can see from the declining numbers in the third column of the accompanying table, electricity does indeed exhibit diminishing returns: the marginal product of each additional kilowatt of electricity is less than that of the previous kilowatt.

Quantity of Electricity (kilowatts)	Quantity of Ice (pounds)	Marginal Product
0	0	---
1	1,000	1,000
2	1,800	800
3	2,400	600
4	2,800	400

   c. Suppose a 50% increase in the size of the fixed input increases output by 100% for any given amount of the variable input. What is the fixed input now? Construct a table showing the quantity of output and marginal product in this case.

   A 50% increase in the size of the fixed input means that Bernie now has a 15-ton machine. So the fixed input is now the 15-ton machine. Since it generates a 100% increase in output for any given amount of electricity, the quantity of output and marginal product are now as shown in the accompanying table.

Quantity of Electricity (kilowatts)	Quantity of Ice (pounds)	Marginal Product
0	0	---
1	2,000	2,000
2	3,600	1,600
3	4,800	1,200
4	5,600	800

2. Alicia's Apple Pies is a roadside business. Alicia must pay $9.00 in rent each day. In addition, it costs her $1.00 to produce the first pie of the day, and each subsequent pie costs 50% more to produce than the one before. For example, the second pie costs $1.00 X 1.5 = $1.50 to produce, and so on.

   a. Calculate Alicia's marginal cost, variable cost, average total cost, average variable cost, and average fixed cost as her daily pie output rises from 0 to 6. (Hint: The variable cost of two pies is just the marginal cost of the first pie, plus the marginal cost of the second, and so on.)

As shown in the accompanying table, the marginal cost for each pie is found by multiplying the marginal cost of the previous pie by 1.5. Variable cost for each output level is found by summing the marginal cost for all the pies produced to reach that output level. So, for example, the variable cost of three pies is $1.00 + $1.50 + $2.25 = $4.75. Average fixed cost for Q pies is calculated as $9.00/Q since fixed cost is $9.00. Average variable cost for Q pies is equal to variable cost for the Q pies divided by Q; for example, the average variable cost of five pies is $13.19/5, or approximately $2.64. Finally, average total cost can be calculated in two equivalent ways: as TC/Q or as AVC + AFC.

Quantity of Pies	Marginal Cost	Variable Cost	Average Fixed Cost	Average Variable Cost	Average Total Cost
0	---	$0.00	---	---	---
1	$1.00	$1.00	$9.00	$1.00	$10.00
2	$1.50	$2.50	$4.50	$1.25	$5.75
3	$2.25	$4.75	$3.00	$1.58	$4.58
4	$3.38	$8.13	$2.25	$2.03	$4.28
5	$5.06	$13.19	$1.80	$2.64	$4.44
6	$7.59	$20.78	$1.50	$3.46	$4.96

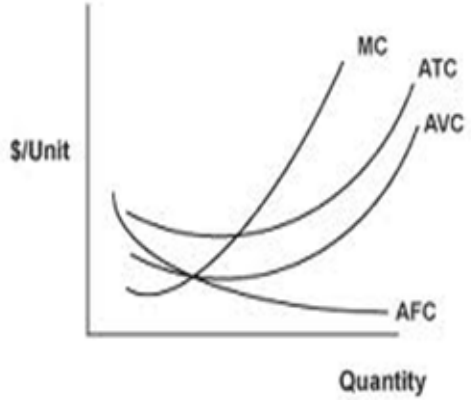
   b. Indicate the range of pies for which the spreading effect dominates and the range for which the diminishing returns effect dominates.

The spreading effect dominates the diminishing returns effect when average total cost is falling: the fall in AFC dominates the rise in AVC for pies 1 to 4. The diminishing returns effect dominates when average total cost is rising: the rise in AVC dominates the fall in AFC for pies 5 and 6.

   c. What is Alicia's minimum-cost output? Explain why making one more pie lowers Alicia's average total cost when output is lower than the minimum-cost output. Similarly, explain why making one more pie raises Alicia's average total cost when output is greater than the minimum-cost output.

Alicia's minimum-cost output is 4 pies; this generates the lowest average total cost, $4.28. When output is less than 4, the marginal cost of a pie is less than the average total cost of the pies already produced. So making an additional pie lowers average total cost. For example, the marginal cost of pie 3 is $2.35, whereas the average total cost of pies 1 and 2 is $5.75. So making pie 3 lowers average total cost to $4.58, equal to (2 X $5.75 + $2.25)/3. When output is more than 4, the marginal cost of a pie is greater than the average total cost of the pies already produced. Consequently, making an additional pie already produced. Consequently, making an additional pie raises average total cost. So, although the marginal cost of pie 6 is $7.59, the average total cost of pies 1 through 5 is $4.44. Making pie 6 raises average total cost to $4.96, equal to (5 X $4.44 + $7.59)/6.

3. Draw a correctly labeled graph showing a firm with an upward sloping *MC* curve and typically shaped *ATC*, *AVC* and *AFC* curves.



4. Marty's Frozen Yogurt is a small shop that sells cups of frozen yogurt in a university town. Marty owns three frozen-yogurt machines. His other inputs are refrigerators, frozen-yogurt mix, cups, sprinkle toppings, and, of course, workers. He estimates that his daily production function when he varies the number of workers employed (and at the same time, of course, yogurt mix, cups, and so on) is as show in the accompanying table.

   a. What are the fixed inputs and variable inputs in the production of cups of frozen yogurt?

The fixed inputs are those whose quantities do not change as the quantity of output changes: frozen-yogurt machines, refrigerators, and the shop. The variable inputs are those whose quantities do change as the quantity of output changes: frozen-yogurt mix, cups, sprinkle toppings, and workers.

   b. What is the marginal product of the first worker? The second worker? The third worker? Why does the marginal product decline as the number of workers increases?

The marginal product, *MPL*, of the first worker is 110 cups. The *MPL* of the second worker is 90 cups. The *MPL* of the third worker is 70 cups. The *MPL* of labor declines as more and more workers are added due to the principle of diminishing returns to labor. Since the number of frozen-yogurt machines is fixed, as workers are added there are fewer and fewer machines for each worker to work with, making each additional worker less and less productive.

   c. Now assume that Marty pays each of his workers $80 per day. The cost of his other variable inputs is $0.50 per cup of yogurt. His fixed cost is $100 per day. Complete the chart below, calculating the variable cost, total cost, marginal cost, average variable cost (AVC), average fixed cost (AFC), and average total cost (ATC) per cup. How many cups of frozen yogurt are produced when ATC is minimized?

Marty's variable cost, *VC*, is his wage cost ($80 per worker per day) and his other input costs ($0.50 per cup). His total cost, *TC*, is the sum of the variable cost and his fixed cost of $100 per day. The answers are given in the accompanying table.

Quantity of Frozen Yogurt (cups)	Variable Cost	Fixed Cost	Total Cost	Marginal Cost (per cup)	*AFC* (per cup)	*AVC* (per cup)	*ATC* (per cup)
0	$0	$100	---	---	---	---	---
110	(1 worker X 80) + (110 cups X 0.5) = $135	$100	$235	$1.23	$0.91	$1.23	$2.14
200	$260	$100	$360	$1.39	$0.50	$1.30	$1.80
270	$375	$100	$475	$1.64	$0.37	$1.39	$1.76
300	$470	$100	$570	$3.17	$0.33	$1.57	$1.90
320	$560	$100	$660	$4.50	$0.31	$1.75	$2.06
330	$645	$100	$745	$8.50	$0.30	$1.95	$2.26

Average total cost is minimized when 270 cups of yogurt are produced. At lower quantities of output, the fall attributable to the spreading effect dominates changes in average total cost. At higher quantities of output, the rise attributable to the diminishing returns effect dominates changes in average total cost.

   d. What principle explains why the *AFC* declines as output increases? What principle explains why the *AVC* increases as output increases? Explain your answers.

AFC declines as output increases due to the spreading effect. The fixed cost is spread over more and more units of output as output increases. *AVC* increases as output increases due to the diminishing returns effect. Due to diminishing returns to labor, it costs more to produce each additional unit of output.